Effect of signal amplitude on magnetic depth estimations

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The magnetic signal generated over any source can be analyzed in terms of the two basic components of any wave: frequency (λ) and amplitude (A). With respect to magnetics, the frequency will be a function of the magnetic source body depth and geometry, while the amplitude is a function of magnetization intensity (magnetic susceptibility and natural remanent magnetization, NRM, if present). Many processing and interpretation methods developed over the last 50 years take advantage of the intrinsic relationship between frequency and depth to generate a variety of depth-estimation routines. Furthermore, many methods are independent of magnetic susceptibility (and therefore amplitude) contrasts since the methods incorporate some sort of mathematical expression that nulls the effect of varying susceptibilities.

One such approach is the "tilt-depth method," which has been shown to work quite reliably on vertical-sided prisms in ambient magnetic fields that are not complex (vertical, no remanence). This method estimates the depth to top of the source body by measuring the physical distance between tilt-angle pairs, with particular emphasis on the locus of the complementary 0° and ±45°pairs. These physical distances remain the same irrespective of magnetic susceptibility due to an integrated horizontal and vertical ratio. However, what is the influence on depth estimates when they are calculated on anomalies of identical frequency but different amplitudes? Here we assess this question through application of varying amplitudes to synthetic magnetic models focusing on one depth estimation routine, tilt depth.

Tilt-depth method

The concept of tilt angle is simply a normalized ratio between the vertical and horizontal derivatives of a potential field signal. Tilt angle, first introduced by Miller and Singh (1994), has since been defined as:

$$\theta = \tan^{-1}\left(\frac{\partial M/\partial z}{\partial M/\partial h}\right) \tag{1}$$

$$\text{where} \quad \frac{\partial M}{\partial h} = \sqrt{\left(\frac{\partial M}{\partial x}\right)^2 + \left(\frac{\partial M}{\partial y}\right)^2} \tag{2}$$

$$\text{and} \quad \frac{\partial M}{\partial x}, \frac{\partial M}{\partial y}, \frac{\partial M}{\partial z} \tag{3}$$

are first-order derivatives of the magnetic field (M) in the directions x, y, and z.

All resultant values will be between −90° and +90° due to the tilt angle being an inverse trigonometric function (arctan). Since tilt angle's introduction in 1994, its capabilities as being a good source edge-detection routine and delineation of source body orientation have been demonstrated by numerous authors (Pilkington and Keating, 2004). Similar to results produced by vertical derivatives, tilt angle will produce a zero value over or near the source edges, with positive values over the source and negative values outside the source. However, unlike vertical derivatives, tilt angle is insensitive to source depth. Normalization produced by the division serves to act as an automatic gain control which results in the tilt-angle method being equally capable of resolving deep and

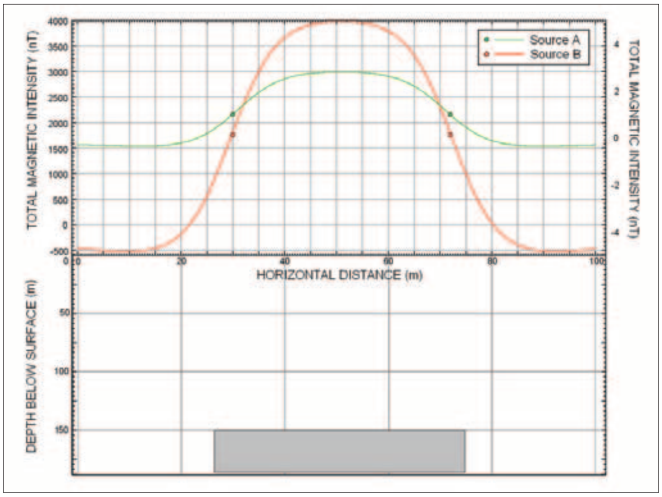

Figure 1. *TMI anomaly generated by two identical sources of different magnetizations at a depth of 150 m. Points of inflection (•) indicate source edges. Magnetic intensity for source A and source B are shown along the right and left vertical axes respectively, as they are at different scales to emphasize signal geometry.*

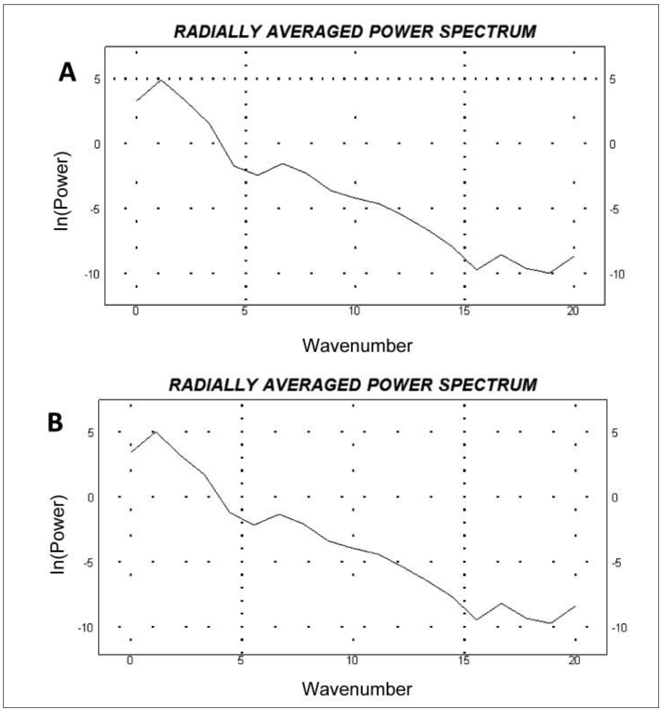

Figure 2. *Power spectrums of source A and source B. Both power spectrums exhibit near identical frequencies, which means any variations in depth estimations would be attributed to the difference in magnetization.*

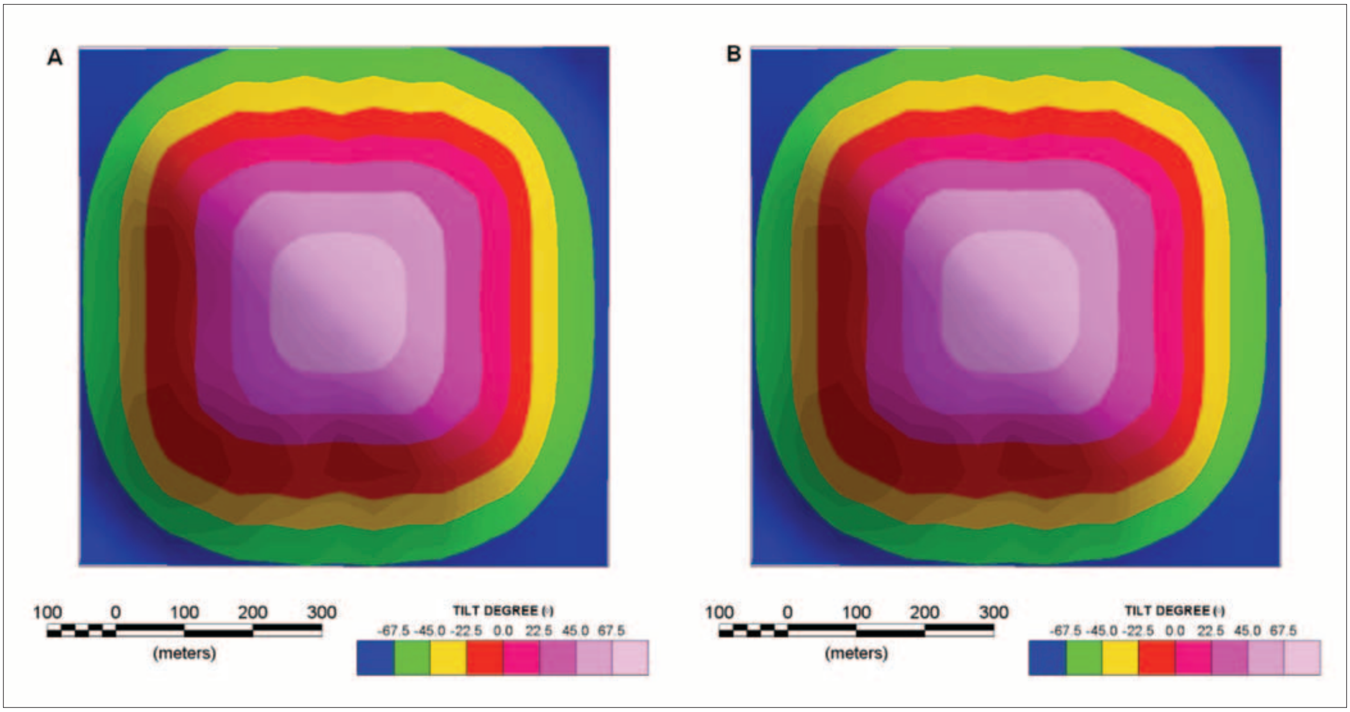

Figure 3. *Tilt angles of source A and source B. Qualitative comparison shows strong agreement.*

shallow sources.

Salem et al. (2007, 2008) showed that tilt angle can be used as a simple method to determine the depth to top of source. This concept known as the "tilt-depth method" incorporates a relationship between the horizontal location of a contact (*h*) and the depth to top of source ($\boldsymbol{z}_T$):

$$\theta = \tan^{-1}\left(\frac{h}{z_T}\right) \tag{4}$$

Reported results using this method have shown incredible accuracy for vertical parallel-sided sources in a vertical ambient field, or having undergone reduction-to-pole (RTP). According to Equation 4, when the results of tilt angle are calculated and contoured, the physical distance between −45°and +45° (2*h*) is equivalent to twice the depth to top (2$\boldsymbol{z}_T$). The physical distance between 0° and ±45° will remain the same regardless of the magnetization. This is courtesy of the integrated horizontal and vertical ratio in Equation 2.

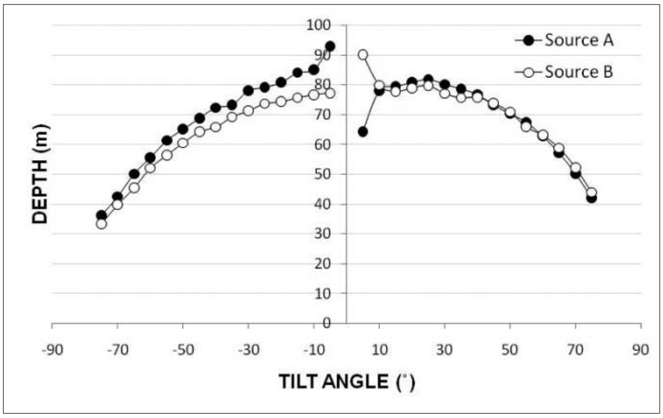

Figure 4. *Tilt-depth results for source A and source B. Computation of z_T for all negative tilt-angle contours results in negative depths. Therefore the absolute value has been taken for all depths calculated using negative tilt angles. The calculated depths for both data sets are nearly identical in both trend and values. The trend exhibits a declining depth value (shallower) with increased tilt angle. For the sake of this article, the trend has been disregarded for the time being as it is identical in both data sets and does not affect the comparison between the two.*

Synthetic model

The most effective way to maintain frequency but vary the amplitude is through varying the magnetization of the source. This can be accomplished through varying magnetic susceptibilities or NRM, although no NRM is being considered here for simplicity. Figure 1 shows the synthetic magnetic anomaly in profile format for two vertical-sided sources both with the same geometry: length = 400 m, width = 400 m, and thickness = 40 m. One source has a magnetic susceptibility of 0.001 SI (source A); the second source has a magnetic susceptibility of 1.4 SI (source B). This corresponds to greater than three orders of magnitude difference between the two susceptibilities. Both synthetic sources are placed in a vertical ambient field of 60,000 nT, eliminating any subsequent possible contamination associated with applying an RTP filter. A line spacing of 100 m and sample spacing of 10 m were used, with the sensor treated a ground surface. The two sources were initially placed at a depth to top of 150 m where the total magnetic intensity (TMI) signal generated is a broad positive peak over the source, the ideal and most simple scenario for synthetic modeling. The position at which the vertical derivative of the TMI changes sign (in

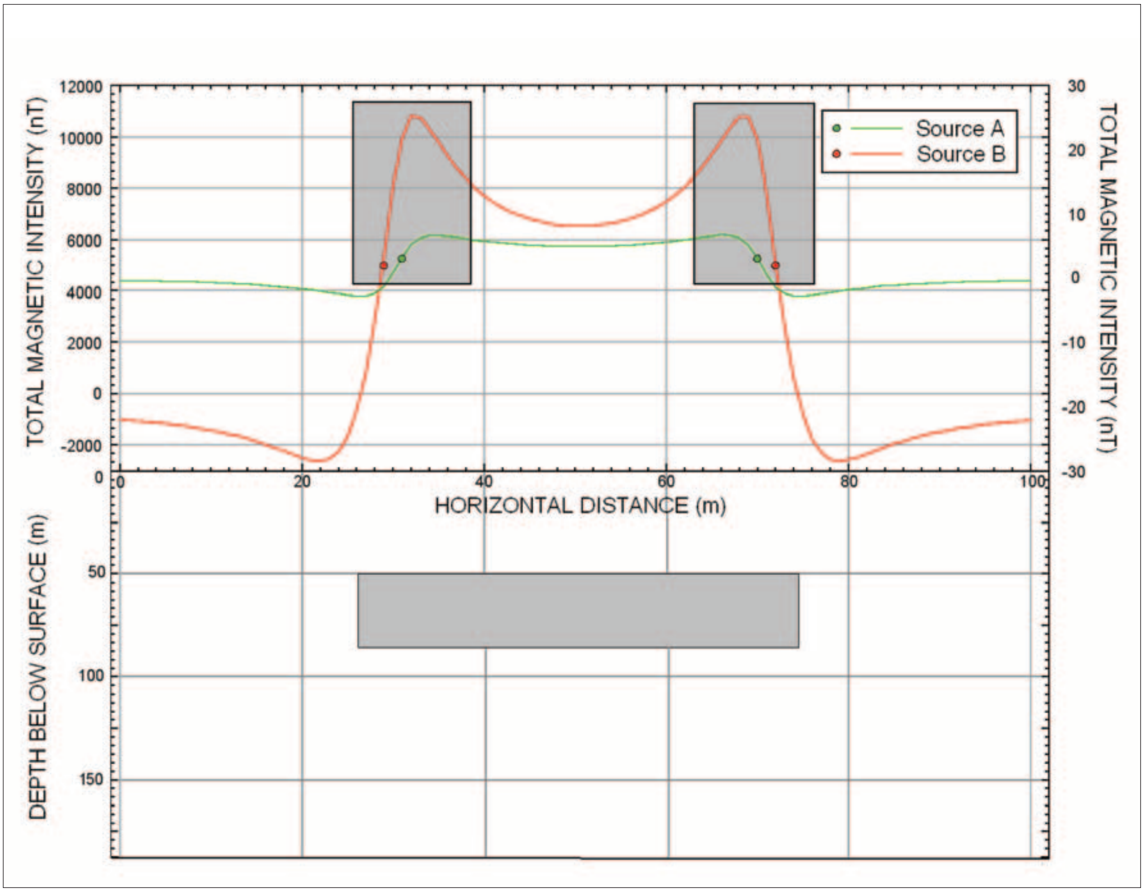

Figure 5. *TMI anomaly generated by two identical sources of different magnetizations at a depth of 50 m. Points of inflection (•) indicate supposed source edges. Magnetic intensities for source A and source B are shown along the right and left vertical axes, respectively, as they are at different scales to emphasize signal geometry.*

profile format) is equivalent to the point of inflection where the distance between two consecutive points of inflection will be equivalent to ½ λ. This location is also equal to the edge of the source. As seen in Figure 1, the distance between inflection points, and therefore frequency, is nearly identical for both sources. Identical frequency content in both signals generated by the two sources can be corroborated through an analysis of their respective power spectrums calculated from the grid data (Figure 2). Subsequently, each TMI data set was gridded using a minimum curvature interpolation scheme and a grid cell size half the line spacing (50 m). This permits the calculation of tilt angle in grid format from the associated TMI data for each source (Figure 3). Contour locations were calculated for every 5° increment of tilt angle (i.e., 5°, 10°, 15°, ...). The computed tilt-depths based on Equation 4 can be seen in Figure 4. The depth solutions have an average value of 68.97 m ± 14.21 and 66.55 m ± 13.61 for source A and source B, respectively—an average depth difference of 2.52 m or 2% of the actual depth (150 m). It is important to note that the physical distances between contours were computed strictly along the central part of source edges since values near source corners can be considered unpredictable due to the abrupt change in magnetic direction. It is accepted that all processing is occurring in a 2D space; however, corners involve more complicated processing since they involve 3D space. Based on the results in Figures 3 and 4, varying amplitude does not have an effect; both source A and source B provided identical depths. This supports the effectiveness

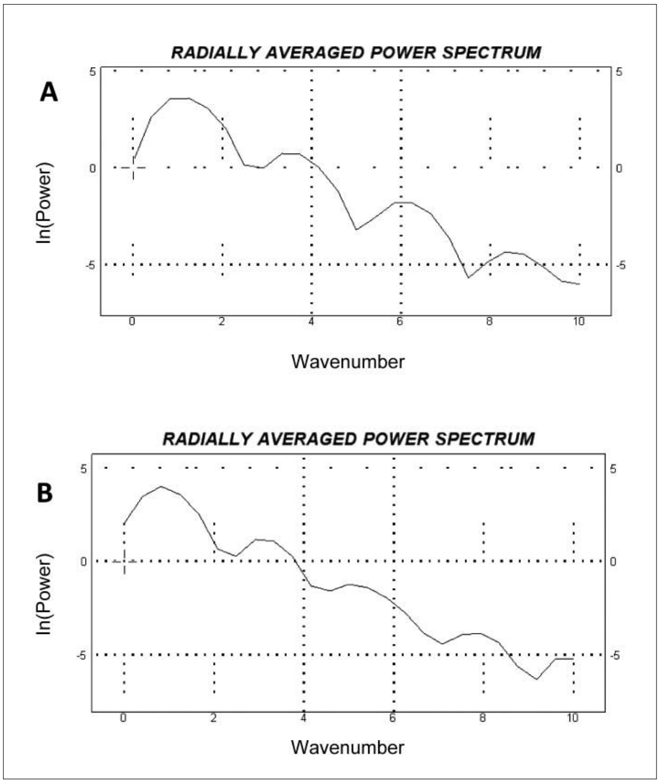

Figure 6. *Power spectrums of source A and source B. The power spectrums do not exhibit identical frequencies, which means any variations in depth estimations could be attributed to either a difference in frequency or amplitude. This causes ambiguity in any depth solutions.*

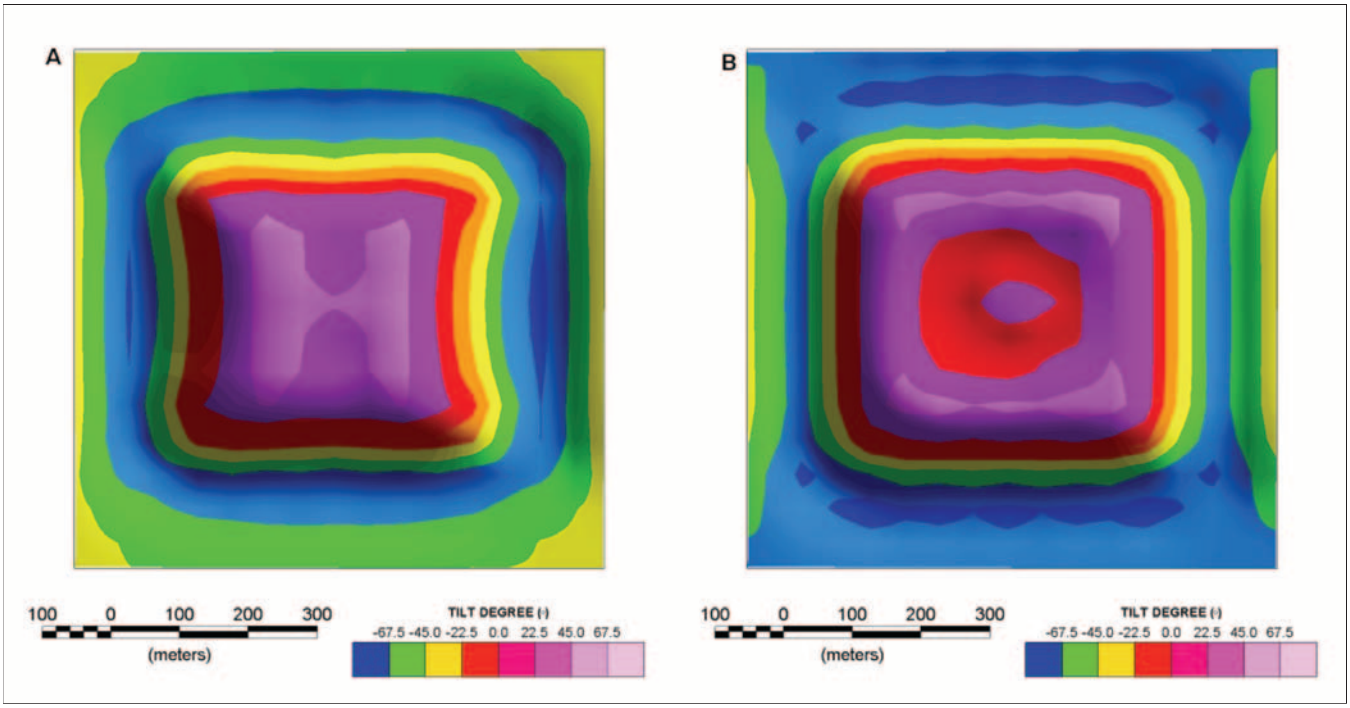

Figure 7. *Tilt angle of source A and source B. Qualitative comparison shows the occurrence of source-edge effects in the tilt-angle calculations.*

of the vertical to horizontal derivative ratio in Equation 1.

Tilt-depth method in this case has not provided the correct depth (150 m). This is due to an orientation parameter effect (magnetic field declination relative to the source edge) which we will report in a separate article. For the interest of this article, the primary objective is whether the calculated depths significantly change with differing magnetization and not whether the calculated depths are accurate. This being said, the depth estimates under these conditions will be the same regardless of the source's magnetization.

Complications

In most instances it is the magnetic contrasts associated with the upper surface of a source body that will dominate the observed magnetic signal. However, under specialized circumstances (i.e., near-surface and/or high magnetization), the magnetic signal will include a contribution from the lower surface of the source body. Based on the magnetic susceptibilities of both the source A and source B synthetic models and their dimensions, the interference between depth to top and depth to bottom occurs 50 m below the surface (Figure 5). The interference between the depth to top and depth to bottom is manifested in the form of "edge effects" (Figure 5). By calculating the power spectrums, it is shown that both data sets in fact do not have identical dominant frequencies as they did in the above model (Figure 6). The TMI is gridded and the tilt-depth method applied to determine a set of depth solutions (Figure 7). The results for calculated tilt depth for every 5th tilt angle can be seen in Figure 8. Source A has a tilt-depth average of 50.11 m ± 8.84 and source B has an average tilt depth of 30.06 m ± 4.16. This results in an apparent source depth difference of 20.05 m, which is 40% of the actual depth (50 m). This difference is one order of magnitude greater than the difference seen in the deep source model. This suggests that tilt depth does not work as effectively for near-surface sources. The power spectrums (Figure 6) reveal different frequencies; therefore, the computed depth differences may be generated by either amplitude or frequency differences. This introduces some ambiguity into all solutions.

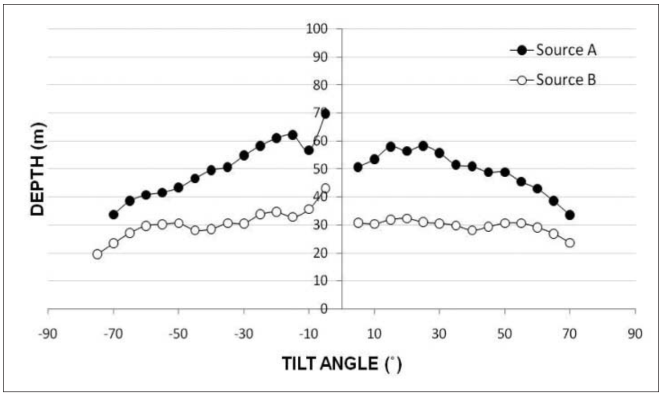

Figure 8. *Tilt-depth results for source A and source B. Computation of z_T for all negative tilt-angle contours results in negative depths. Therefore the absolute value has been taken for all depths calculated using negative tilt angles. The calculated depths for both data sets are similar in trend but different in values. Once again both data sets exhibit a declining trend with increased tilt angle. Since the trends are nearly identical in both data sets, they have been ignored.*

Typically, the geometry of magnetic signal is dominated by the depth to top and minimally by the depth to bottom (Spector and Grant, 1970). Since a magnetic field will decay at a rate inversely proportional to source-signal distance cubed, often the signal generated by the bottom of the source has decayed before reaching the sensor. This is true for most cases and as such the depth to bottom is typically not taken into consideration. In isolated instances, such as with very thin, shallow bodies, the depth to bottom can have a significant influence on the resultant anomaly. In this situation, the signal produced by the bottom of the source is readily detectable by the sensor. Since the TMI signal at any point represents the summation of all sources within proximity, The ridges and troughs (anomalies) which characterize all TMI maps are a record of the summation (interference) of the field generated by various sources. Applying an analytic routine to this type of mixed source data will produce many solutions that are not geologically meaningful. This emphasizes the importance of the application of filtering routines prior to any sort of processing and interpretation.

Conclusions

Many different depth-estimation routines are currently being implemented on magnetic survey data, and all take advantage of the relationship between the wave parameter frequency and depth. As demonstrated in this article, another wave parameter, amplitude, has very little influence on the calculated depths under ideal situations (nondipping source in a vertical ambient field) when using tilt-depth method. Furthermore, most depth-estimation routines incorporate some mathematical function into their fundamental equations (such as Equation 1 for tilt depth) that allow depth estimations to be identical regardless of source magnetization (and thus amplitude). It has been shown that this is true under the application of tilt depth to a synthetically generated model at a depth of 150 m.

That being said, depth estimations do not work under all conditions. The reason why tilt depth fails for shallow, highly magnetic sources is that like most semi-automatic interpretation routines, it involves gradients of the magnetic field, with the primary assumption that the data within an individual window define a single isolated magnetic anomaly associated with a single geological source. However, these conditions are rarely met. In this case, although there is a single geological source, a mixed magnetic signal was generated due to the interaction between the signal generated by the source depth and that of the source bottom. This denotes why processing routine limitations need to be considered. Additionally, a future consideration is how depth-estimation routines are affected by a varying topography. This added parameter can cause a dampening or amplifying effect on the recorded magnetic signal depending on where a source is relative to the topographic or airborne draped surface. TLE

References

Spector, A. and F. Grant, 1970, Statistical models for interpreting aeromagnetic data: Geophysics, **35**, 293–302.

Li, X., 2003, On the use of different methods for estimating magnetic depth: The Leading Edge, **22**, 1090–1099.

Phillips, J., R. Hansen, and R. Blakely, 2007, The use of curvature in potential-field interpretation: Exploration Geophysics, **38**, 111–119.

Pilkington, M. and P. Keating, 2004, Contact mapping from gridded magnetic data—a comparison of techniques: Exploration Geophysics, **35**, 306–311.

Verduzco, B., J. D. Fairhead, C. Green, and C. MacKenzie, 2004, New insights into magnetic derivatives for structural mapping: The Leading Edge, **23**, 116–119.

Miller, H. and V. Singh, 1994, Potential field tilt—a new concept for location of potential field sources: Applied Geophysics, **32**, 213–217.

Salem, A., S. Williams, J. D. Fairhead, D. Ravat, and R. Smith, 2007, Tilt-depth method: A simple depth estimation method using first-order magnetic derivatives: The Leading Edge, **26**, 1502–1505.

Salem, A., S. Williams, D. Fairhead, R. Smith, and D. Ravat, 2008, Interpretation of magnetic data using tilt-angle derivatives: Geophysics, **73**, no. 1, L1–L10.

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